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ABSTRACT

The increasing availability of the computer in the classroom has created a need for informed and creative instructional software. Two areas of cognitive research in mathematics and science learning have implications to instruction and, consequently, to educational software design. These are research on misconceptions and on differences between expert and novice problem solvers. This paper treats these areas in two major sections. The first section provides a broad overview of the misconception literature and of the instructional implications associated with this body of research. Also included in the first section is a description of a computer-based approach designed to teach students how to translate algebraic word problems into equations. The second major section provides an overview of the differences between experts and novices and discusses a computer-based approach in which students are constrained to analyze physics in expert-like ways. The computer-based example used relates to the analysis of classical mechanics problems. A 31-item list of references concludes the document. (TW)

Computer-Based Methods for Promoting Thinking in Physics and Algebra: Incorporating Cognitive Research Findings into Software Design.

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The increasing availability of the computer in the classroom has created a need for informed and creative instructional software. We think that the field of cognitive research in mathematics and science problem solving processes can contribute significantly to the development of such software. Until recently, the fields of cognitive research and educational software design proceeded along fairly independent, non-crossing paths. Despite the fact that an increasing amount of this research has direct instructional implications, the results of cognitive studies are rarely published in journals read by educators or practitioners. Similarly, the majority of math and science educational software has been developed by expert programmers who have limited knowledge of pedagogy or learning theory. The result is software that is often flashy but does not enhance higher order thinking skills. Given this situation, it is not surprising that over 90% of the math/science educational software on the market is of the "drill and practice" variety.

Two areas of cognitive research in math and science learning bear implications to instruction, and consequently to educational software design. These are research on misconceptions and on differences between expert and novice problem solvers. This chapter treats these areas in two major sections. In the first section we provide a broad overview of the misconception literature and of the instructional implications associated with this body of research; we then describe a computer-based approach designed to teach students how to translate algebraic word problems into equations. The second section provides an overview of the differences between experts and novices, and discusses a computer-based approach in which students are constrained to analyze physics problems in expert-like ways.

CONSIDERING MISCONCEPTIONS IN THE DESIGN OF SOFTWARE

Misconceptions implicitly influence how we learn and think about physical and mathematical phenomena. However, this has not always been recognized. Many educators assume that if the presentation is clear enough, the students will learn. This view has changed. Our current understanding is that students possess naive theories that they use in dealing with physical and mathematical phenomena. Further, these naive theories are often incorrect and actually interfere with students' ability to understand some subtle physical and mathematical concepts.

Naive theories develop as a natural result of our attempts to explain, categorize and order world events. The construction of these theories is active, although often unconscious (Resnick, 1983), and is crucial to all learning. The problem is that naive theories tend to be incomplete, fragmented, and contain misconceptions which interfere with learning.

Misconceptions have been observed in all scientific fields that have been investigated to date, including physics (Clement, 1982a; Fredette and Clement, 1981; McCloskey, Caramazza, and Green, 1980; McDermott, 1984), probability and statistics (Tversky and Kahneman, 1977; Pollatsek, Lima and Well, 1981), and elementary mathematics and algebra (for a compendium, see Benander and Clement, 1986). It might appear that a teacher armed with a knowledge of likely misconceptions could present material in a way that eradicates students' existing misconceptions and supplants them with correct notions. However, simply telling students that their conceptual understanding is wrong or incomplete is not sufficent to permanently eradicate most misconceptions (McDermott, 1984; Resnick, 1983). Misconceptions, by virtue of the fact that students have spent time and energy constructing them, are deeply-seated and difficult to dislodge. Even students who display appropriate understanding of concepts immediately following a presentation and do well in course tests often display evidence of misconceptions a short time later.

This research implies that teaching can be ineffectual if it does not address incorrect, deeply-held student beliefs. We believe that in order to increase the effectiveness of instruction, the educational process needs to become more bidirectional. In other words, instructors should attempt to become more cognizant of their students' misconceptions and address these openly during the course of instruction, rather than simply presenting information. This instructional approach is implemented in a computer program, to be described, designed to teach students how to translate word problems into equations. We begin with a discussion of the difficulties and misconceptions that students encounter.

Misconceptions Displayed in Translating Algebra Word Problems

Translating English statements that express a relationship between two variables into an equivalent algebraic equation presents difficulty for many students (Clement, 1982b; Clement, Lochhead and Monk, 1981; Lochhead, 1980; Lochhead and Mestre, in press; Mestre, 1987; Mestre and Gerace, 1986; Mestre, Gerace and Lochhead, 1982; Mestre and Lochhead, 1983; Rosnick 1981, Rosnick and Clement 1980). An example of a problem that students have difficulty translating is the "students and professors" problem:

> Write an equation using the variables S and P to represent the following statement: "There are six times as many students as professors at this university." Use S for the number of students and P for the number of professors.

Engineering students failed this task at a rate of 37%, while non-science majors failed at the rate of 57% (Clement, et al., 1981). Hispanic engineering students failed at the rate of 58% (Mestre, Gerace and Lochhead, 1982). By far the most common error made in this problem was to write 6S=P instead of the correct equation, 6P=S. For obvious reasons, this error is called the "variable-reversal error."

If the problem is slightly more complicated, the error rate ranges between 60% and 80%, as in:

> Write an equation using the variables C and S to represent the following statement: "At Mindy's restaurant, for every four people who ordered cheesecake, there were five who ordered strudel." Let C represent the number of cheesecakes ordered and let S represent the number of strudels ordered.

Again, the common error is the variable reversal error, 4C=5S.

The variable-reversal error cannot be attributed to either of two fairly obvious sources. It is not due to a simple misreading of the problem: interviews with students solving the "students and professors" problem reveal that they recognize that there are more students than professors. Neither can the difficulty be attributed to a lack of fluency in algebraic manipulation skills, since diagnostic tests did not reveal any deficiencies in this area (see Lochhead and Mestre, in press).

There appear to be two major reasons why students make the variable-reversal error. The first is a strong proclivity to translate word problems using a left-to-right word-order matching technique. Thus, "six times as many students" becomes "6S," "professors" becomes "P," yielding "6S=P." The second reason stems from an inability to make appropriate distinctions between variables and labels. Students who write the variable-reversed equation interpret the symbols "S" and "P" to mean the labels "students" and "professors," rather than the variables, "number of students" and "number of professors;" to these students, the variable-reversed equation, 6S=P, stands for "six students for every one professor." This apparently minor distinction is crucial in mathematics.

The variable-reversal error is not unique to the American educational system. In studies including Japanese college students majoring in the social sciences, Israeli Hebrew-English bilingual pre-college and college students, and Fijian Hindi-English and Fijian-English bilingual college education majors, the same error patterns were found (Eylon, Ikeda and Kishor, 1985; Mestre and Lochhead, 1983).

In sum, students may harbor misconceptions despite an apparent acceptance and command of newly presented concepts. In time, it is possible for the newly acquired knowledge to decay and misconceptions to resurface. When this happens, old misconceptions and new knowledge can be in conflict, thereby inhibiting the learning process. If an old misconception and newly learned concept can be counterpoised so that resolution of an appreciated conflict occurs, then the new concept should replace the old misconception rather than coexisting with it. Thus, the identification of a misconception and subsequent juxtaposition to the correct concept can help the student construct appropriate understanding. However, students must actively participate in this process; new ideas will not be linked to existing ideas unless the student is actively engaged in interweaving knowledge and premises, and in seeking relationships and inconsistencies.

A Computer-Based Instructional Approach

The computer software that we have developed is intended to teach students how to translate simple algebraic word problems into equations. We have attempted to incorporate findings from the cognitive research previously described into the design of our software, in an attempt to identify and to help the student overcome common misconceptions. It is our hope that describing this approach will encourage other software designers to incorporate findings from cognitive research into their instructional design.

The software is designed to evaluate three phases of understanding: 1) qualitative, 2) quantitative, and 3) conceptual. We begin by presenting a situation describing a mathematical relationship and ascertain whether or not the student possesses a qualitative understanding:

> There are six times as many students as professors at Baker University.
> Are there more students or professors at Baker University?

It is our experience that this question will not cause difficulty for many students, except for those bilingual students who have difficulties with the English language.

Next we probe for quantitative understanding, by asking: "Suppose that there are 100 professors at Baker University. How many students would there be?" Again, most students can readily state the answer, namely, that there would be 600 students.

Finally, we probe for conceptual understanding by asking the student to write an equation that depicts the situation in the problem:

> There are six times as many students as professors at Baker University. Write an equation to represent this situation using S to stand for the number of students and P to stand for the number of professors.

Here, we look for specific erroroneous responses that are indicative of misconceptions. Nearly all erroneous responses fall into one of three possible categories: 1) the variable-reversal error, 6S=P, 2) the "total population error," 6S+P=T, and 3) the "equation-impossible error," 6S/P. Students who write 6S+P=T claim that they are relating the number of students, professors, and the total (T) student-teacher population. In addition to committing the variable-reversal error these students are also defining an extraneous variable. Those who commit the "equation-impossible error," 6S/P, claim that one can never write an equation because there are more students than professors and thus writing an "=" in an equation would incorrectly imply that the two populations are equal.

We illustrate how our computer-based approach deals with misconceptions with the specific case of the variable-reversal error, 6S=P. Our goal is to reveal a contradiction between the student's equation, and statements previously made by the student in the qualitative and quantitative phases of understanding. We begin by asking the student to check his or her equation by substituting a given number for one of the variables, such as S=600, and evaluating the other variable, P. The student is likely to give one of two answers: 1) a correct substitution into 6S=P, or P=3600, or 2) the correct answer according to the problem situation, namely P=100. In the former case, the contradiction is revealed by pointing out the inconsistency between the statement that there are more students than professors and the opposite result given by the equation. In the latter case, the student clearly does not employ an appropriate substitution process, but cues on the fact that, logically, S must be larger than P. In this case, we actually substitute S=600 into 6S=P and show that the result is P=3600, thereby revealing the contradiction.

The approach is meant to resemble a computer-directed "Socratic dialogue," during which the student is seldom told the correct answer. Instead, we ask questions that attempt to reveal any contradiction between the student's answer and the problem statement, and then guide the student, through additional probing questions, toward a resolution of the contradiction. We feel that this approach allows students to grapple with their own misconceptions so that they actively try to extirpate and supplant them with the appropriate understanding. If a student has difficulty coming to a resolution, 'ere is a "Help" section available. This "Help" section addresses several "thorny" topics, such as "When is a mathematical expression an equation?," "What is the difference between a variable and a label?," and "How do you know what operation to use?" A more direct tutorial approach is used in this section of the software.

The concept of a dialogue is enhanced by the fact that questions may be posed that require a free-form response, rather than a single letter, or multiple choice selection. With this form of computer-student dialogue the student types in a free-form answer from the keyboard. For example, a response to, "Are there more students or more professors at Baker University?", might be "there are more students" or "they are equal." The software is capable of analyzing these responses for correctness. This feature gives the user more control thereby increasing the possibility for interaction.

If the student is not fluent in English, the likelihood of a meaningful dialogue is diminished. Hence, the program contains an option that allows Hispanic students (a sizeable minority in some areas of the United States) who have difficulty with English to toggle back and forth between an English and a

Spanish presentation. This option should hopefully diminish the likelihood of misinterpretations, and assist Hispanics in raising their low level of participation in mathematics and math-related careers (Mestre, 1986).

This instructional approach is not limited to the computer and could easily be used within a classroom setting. The teacher would need some knowledge of possible misconceptions in order to pose probing questions aimed at revealing misconceptions that the students might possess. When misconceptions are identified, the teacher could engage the class in a Socratic-like dialogue, or serve as moderator while different factions of the class argue their point of view. In either case, the ensuing interactions would provide a good environment for exposing and resolving contradictions.

Next we will discuss expert-novice differences in the domain of physics, and describe a problem solving environment in which novices can actively participate in analyzing problems using an expert-like approach.

AN EXPERT APPROACH TO PROBLEM ANALYSIS IN PHYSICS

Expert-Novice Differences in Knowledge Organization and Problem Solving

The consensus of a number of studies in such diverse fields as chess (Chase and Simon, 1973), electrical circuits (Egan and Schwartz, 1979) and computer programming (Ehrlich and Soloway, 1982) is that novices and experts store and use domain-specific knowledge in distinctly different ways. Experts tend to store information in hierarchically structured clusters related by underlying principles or concepts. When attempting to solve a problem in a domain like physics, experts initially focus on the principles and heuristics that could be applied to solve that problem. In contrast, the knowledge base of novices is somewhat amorphous with few interconnections. When solving problems, physics novices focus on the actual equations that could be manipulated to yield an answer (Chi, Feltovich, and Glaser, 1981; Larkin, McDermott, Simon, and Simon, 1980; Mestre and Gerace, 1986).

The expert's knowledge-base has been described by Chi and Glaser (1981) as having: a) more central concepts or conceptual nodes in memory, b) more relations or features defining each node, c) more interrelations among nodes, and d) effective methods for retrieving related nodes. An expert's knowledge-store is thus characterized as dense, containing clusters of related information, whereas the novice's network is sparse with relatively few interrelated clusters. Despite these pronounced differences between experts and novices, studies (Eylon and Reif, 1984; Heller and Reif, 1984) suggest it may be possible to improve the performance of novices in problem categorization and problem solving by using an instructional approach that imposes a hierarchical, expert-like organization on information and problem analysis.

Eylon and Reif (1984) found that undergraduate subjects presented with a physics argument organized in hierarchical form performed significantly better on recall and problem solving tasks than did subjects who had received the same argument non-hierarchically. A second experiment showed that the hierarchical presentation of a set of rules needed to solve a class of physics problems resulted in better performance on subsequent problems than a non-hierarchical presentation of the same set of rules. The results of these studies suggest that the organization of the knowledge imparted in teaching is as important as the knowledge itself, since the organization has an effect on intellectual performance.

Another study by Heller and Reif (1984) suggests that novices can benefit from instruction on expert-like approaches to solving problems. In their study, novices were trained to generate a problem analysis called a "theoretical problem description." These analyses required one to describe a force problem from classical mechanics in terms of concepts, principles and heuristics. When induced to make such analyses, novices substantially improved their ability to construct problem solutions. Heller and Reif point out that this type of analysis is not a naturally occurring phenomenon even among novices who received good grades in a classical mechanics course: These subjects were unable to generate appropriate descriptions of fairly routine problems. Heller and Reif also point out that the abilities to: a) describe a problem in detail before attempting a solution, b) determine what relevant information should go into the analysis of a problem, and c) decide which procedures can be used to generate a problem description and analysis, are skills tacitly possessed by experts but are rarely taught explicitly in physics courses.

Together, the two Reif studies indicate that the performance of novices on several types of tasks can be improved when they are taught using pedagogical approaches designed to reflect expert knowledge organization and behavior. In the next section we describe a hierarchical, computer-based problem analysis tool designed to allow novices to participate in expert-like problem solving activities.

Architecture of a Hierarchical Analysis Tool in Classical Mechanics

The computer-based tool we describe below is designed to help novices conduct an expert-like analysis of classical mechanics problems. We have called it the Hierarchical Analysis Tool (HAT) because the user begins by categorizing the problem on the basis of fundamental principles and concepts, and then answers a set of more specific questions leading to a set of equations that could be used to solve the problem. We emphasize that it is a "tool" because the computer-based environment does not directly teach or tutor the student—it only constrains the user to follow an expert-like analysis of the problem.

The Hierarchical Analysis Tool operates as follows. The user is given a problem written on an index card and asked to categorize the problem by answering some well-defined questions presented via a series of menus. The user categorizes the problem by selecting the one choice that is appropriate for the problem under consideration. The selection made at any menu leads to another menu that is more specific than the previous menu and contains menu selections consistent with the previous selections. The first menu asks the user to select among four fundamental concepts that could be applied to solve the problem: (1) Newton's Second Law or Kinematics, (2) Angular Momentum, (3) Linear Momentum, and (4) Work and Energy. The questions posed in the menus that follow involve expert-like heuristics that could be used in constructing a solution. The final result of the analysis is a set of equations dynamically constructed in accordance with the classification scheme selected.

The best means of understanding the structure and functioning of the Hierarchical Analysis Tool is to consider an example:

PROBLEM 1

A small block of mass M slides along a track having both curved and horizontal sections as shown. The track is frictionless. If the particle is released from rest at height h, what is its speed when it is on the horizontal section of the track?

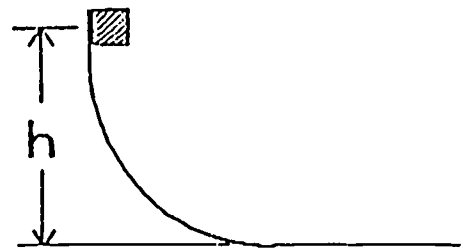


Figure 1 contains the series of menus a· ·enu selections which appropriately analyze Problem 1 (we have placed an asterisk next to the appropriate choice to facilitate discussions). Several features of Figure 1 should be noted. This problem can be solved most easily using work and energy principles, making menu item #4 is the appropriate first selection. As can be observed from Figure 1, second menu level becomes more specific. Explanatory information is provided (enclosed in parentheses) to help the user decipher the choices presented.

Heuristics dictate the choices presented in menu 3: the user is asked to classify the changes in mechanical energy by considering only one body at a time at some initial and final state. In Problem 1, the block starts out with only potential energy and ends up with only kinetic energy, and hence selection #3 is the appropriate choice. The fourth menu asks the user to characterize the changes in kinetic energy, which in this case are comprised purely of changes in translational kinetic energy. The user must then specify the boundary conditions (i.e., conditions at the beginning and end points). This cycle is repeated to describe the changes in potential energy.

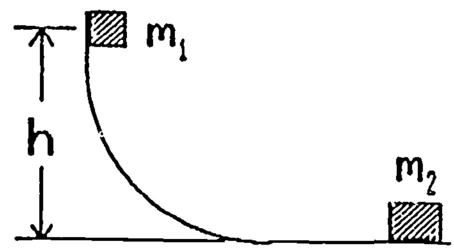
At menu level 8, the user is asked whether there is more than one body in the system. Since there is not, the user is given a summary of the solution path generated thus far, which includes the principle selected initially in a general equation form, as well as the specific equations dictated by the selections made during the analysis. If appropriate selections were made, then the general and specific equations can be combined to generate a correct answer to the problem. For Problem 1, the user would have to manipulate the equations given in menu 9 to obtain the answer, $v=\sqrt{2gh}$.

If the user makes an inappropriate selection at any menu level during the analysis, the end result is a set of equations that is consistent with the classification scheme selected, but inappropriate to use in solving the problem under consideration. Thus, the appropriateness of the final equations depends on the appropriateness of the selections made along the way. The user may become aware of errors he or she committed during the analysis by recognizing that a particular set of menu options, or that the final set of equations, do not fit the problem being analyzed; if this is the case, the user may back up to some previous menu and change a selection. It is possible to back up as many levels as desired. The user does have the option of listing all the menu selections made previous to the current menu in order to determine how far to back up the analysis. Another option is simply to restart the analysis. If any of the terms that appear in a menu are unfamiliar, the user can look up a term in a glossary. The Hierarchical Analysis Tool then returns the user to the analysis without loss of continuity.

At the final menu presented in Figure 1, the user must decide whether or not the analysis of the problem is complete. If the problem has more than one part and requires the application of more than one concept or principle, then the user can opt to continue analysis of the problem. For example, consider the following twist to Problem 1:

PROBLEM #2:

A block of mass m_1 is released from rest at height h on a frictionless track having both curved and horizontal sections as shown. When the block reaches the horizontal section, it collides and sticks to another block of mass m_2. Find the final speed of the two-block system.



Problem 2 must be solved using a sequential application of Work-Energy and Linear Momentum principles. First one needs to obtain the speed of block m_1 when it reaches the horizontal part of the track using conservation of energy, just as in Problem 1. The user would then return to the Main Menu to continue the solution, selecting "Linear Momentum" in order to determine the final speed of the two-block system after the collision. Figure 2 provides the series of menus and choices for the analysis of the "momentum" portion of Problem 2. The end result is two "equation menus" that would be used to solve the problem: that in Figure 1, which allows the computation of the speed of m_1 when it reaches the bottom of the ramp, and that in Figure 2, which allows the computation of the final speed of the two-block system.

In summary, the Hierarchical Analysis Tool is a very rich environment which allows its user to conduct a hierarchical, concept-based, qualitative analysis of a classical mechanics problem that results in a set of equations that could be applied to solve the problem. It is capable of handling the majority of problems encountered in a beginning college level classical mechanics course. We think that a major strength of the Analysis Tool is that it allows students to experience the manner in which an expert would analyze a problem before carrying out a solution strategy. That is, it emphasizes the application of concepts and general strategies to solve problems, rather than the "find-an-equation-to-plug-into" approach used by most physics novices.

We find it regrettable that students generally have little opportunity to observe expert problem-solving behavior during the course of their college education. Even when an expert solves a problem in front of a novice, the expert's presentation is likely to be structured for elegance and clarity, and may bear little resemblance to the expert's normal problem solving behavior. Since cognitive findings indicate that one of the best ways to learn is by doing (Anzai and Simon, 1979), the Hierarchical Analysis Tool provides novices with an opportunity to mimic expert-like problem-solving behavior. It also addresses a common complaint voiced by many students enrolled in physics courses, namely, "I don't even know how to start the problem." The approach emphasized in the Hierarchical Analysis Tool will always let the novice at least start a problem.

CONCLUDING REMARKS

The increasing body of cognitive research in the domains of mathematics and science is beginning to provide invaluable information on how people learn and solve problems in these domains. This body of research is also shedding light on how to make instruction more effective and efficient. Combining cognitive research findings with the power and versatility of computers has opened a new instructional realm which is only now beginning to be explored. We have discussed two examples that illustrate this synergy. The first example from the domain of algebra has a specific instructional agenda consisting of teaching students to translate word problems into equations while at the same time addressing students' misconceptions. The second example from the domain of physics provides the student with a very rich environment in which to actively participate in problem solving activities in ways that resemble the approach taken by experts.

In closing, we would like to point out that the field of cognition and instruction is fairly young. The number of innovative instructional approaches, whether designed to be delivered via computer or via more traditional methods, is increasing. Concomitant with these innovations are a number of important questions that have not begun to be investigated--among the most important of these questions are, "How can cognitive process instruction be best incorporated into our educational system?" and, "What will be the costs, measured both in terms of money and teacher/student morale, of incorporating these approaches into our educational system?"

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FIGURE 1

HIERARCHICAL ANALYZER MENUS & CHOICES FOR PROBLEM 1

1

Which principle applies to this part of the problem solution?

1. Newton's Second Law or Kinematics
2. Angular Momentum
3. Linear Momentum
*4. Work and Energy

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

2

Describe the system in terms of its mechanical energy

*1. Conservative system (conservation of energy)
2. Non-conservative system (work-energy exchange)

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

3

Describe the changes in mechanical energy. Consider only the energy of one body at some initial and final state

1. Change in kinetic energy
2. Change in potential energy
*3. Change in potential and kinetic energies

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

4

Describe the changes in kinetic energy

*1. Change in translational kinetic energy
2. Change in rotational kinetic energy
3. Change in translational and rotational kinetic energies

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

5

Describe the boundary conditions

*1. No initial translational kinetic energy
2. No final translational kinetic energy
3. Initial and final translational kinetic energies

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

6

Describe the changes in potential energy

*1. Changes in gravitational potential energy
2. Changes in spring potential energy
3. Changes in gravitational and spring potential energies

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

7

Describe the boundary conditions

1. No initial gravitational potential energy
*2. No final gravitational potential energy
3. Initial and final gravitational potential energy

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

8

Is there another body in the system which has not been examined?

1. Yes
*2. No

Please enter your selection: __

(B)ackup (M)ain menu (H)elp (Q)uit (L)ist selections

9

The Energy Principle states that the work done on the system by all non-conservative forces is equal to the change in the mechanical energy of the system:

$$W_{nc} = E_f - E_i$$

According to your selections,

$W_{nc} = 0$ (Conservative system: mechanical energy conserved)

$E_f = (1/2\ Mv^2)_{1f}$

$E_i = (Mgy)_{1i}$

Please press any key to continue

10

*** Work and Energy ***

1. Problem solved
2. Return to Main Menu to continue solution
3. Review previous solution screens

Please enter your selection: __

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FIGURE 2

HIERARCHICAL ANALYZER MENUS & CHOICES
FOR SECOND PART OF PROBLEM #2

1

Which principle applies to this part of the problem solution?

1. Newton's Second Law or Kinematics
2. Angular Momentum
*3. Linear Momentum
4. Work and Energy

Please enter your selection: __

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

2

Describe the system in terms of its linear momentum

*1. Momentum conserved (external forces do no work)
2. Momentum not conserved (external force does work)

Please enter your selection: __

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

3

Describe the system at some initial state.

1. One particle
*2. Two particles
3. More than two particles

Please enter your selection: __

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

4

Describe all motion within the system at some initial state

1. No motion
*2. One particle in motion
3. Two particles in motion

Please enter your selection: __

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

5

Describe the system at some final state

*1. One particle
2. Two particles
3. More than two particles

Please enter your selection: __

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

6

Describe all motion within the system at some final state

1. No motion
*2. One particle in motion

Please enter your selection: __

(B)ackup (M)ain menu (G)lossary (Q)uit (L)ist selections

7

The impulse-momentum theorem states that the impulse delivered to a system is equal to the change in momentum of the system

$$\int F_{ext}\,dt = P_f - P_i$$

According to your selections:

$$\int F_{ext}\,dt = 0 \quad \text{(conservation of momentum)}$$

$$P_i = M_1 V_{1i}$$

$$P_f = (M_1 + M_2) V_f$$

Press any key to continue

8

*** Final Menu ***

1. Problem solved
2. Return to Main Menu to continue solution
3. Review previous solution screens

Please enter your selection: __